Response to Dana Center Review – Bridges in Mathematics K-2

by Allyn Fisher, Director of Curriculum Development, Math Learning Center

As one of the authors of the Bridges program, I found the Dana Center Summary of Evidence to be thoughtful, even-handed, and fairly accurate. The evaluation provides us valuable feedback as we continue to bring Bridges into optimal alignment with the Common Core State Standards. However, there are three statements in the summary that we feel are inaccurate and in response I offer the comments below.

A. Under the heading of Standards for Mathematical Practice 3, the reviewers state that, "although [they] found many examples of students sharing and justifying their thinking and critiquing solutions of others, no evidence was found for using non-examples throughout this resource."

I offer three responses to this assertion:

1. Any time students are sharing and justifying their thinking, non-examples or incorrect answers will be volunteered. If teachers are able to "hold the discomfort" of an incorrect response, students will almost invariably find their own errors or correct one another. Here is a simple example from Bridges Grade 2. In Unit 4, Session 25, the teacher introduces a task in which students will circle all the multiples of 3 on a hundreds grid. Before they start, the teacher asks the class,

Will you be circling any even numbers when you count by 3's on this grid?

In the sample dialog that ensues, one of the students responds by saying no. A second student says, "Yes! What about 6 and 12?" A third student then volunteers, "So maybe 100 will get circled."

The teacher does not comment or correct at this point, so the third student's comment serves as a non-example, and an intriguing one at that. Many of the students go into the exercise curious to find out if 100 will actually be circled, and if not, why not.

2. It seems to me that nearly every sorting activity throughout Bridges K-2, and there are many, includes non-examples. In Bridges Kindergarten, for example, there is a simple activity (Session 53) in which the teacher has a shape card in her pocket. The students ask questions to try to determine which shape it is. In order to support the students in their questioning, the teacher has posted a set of shape and shape attribute cards in a pocket chart. As students ask questions (Is your shape red? Is it little? Is it yellow? Does your shape have 4 corners?), cards in the collection are turned face down narrowing the range of possibilities until the shape in the teacher's pocket is identified correctly. Each of the cards turned face down serves as a non-example of the actual shape.

3. There are activities in the program that actually do work on the basis of examples and non-examples. Supplement Set C4 for first grade, for instance, involves the concept of symmetry. Each day of the month, students investigate a 2-D shape pictured on a calendar marker, looking for lines of symmetry. Every third shape is, in fact,

asymmetrical, and the figures that lack symmetry are just as important as the symmetrical shapes in helping students understand the concept.

B. Under the heading of Standards for Mathematical Practice 4, the Dana Center reviewers state that, "In the Grade 2 materials there is some evidence of various models/tools being used, but no evidence was found for students creating and revising mathematical models."

There are actually many opportunities for second graders to create and revise mathematical models throughout the program. Perhaps the easiest place to see this in action is Unit 5, which was predicated on research done by Paul Cobb, Les Steffe, et al, in the mid-90's to the effect that when second graders are allowed to invent their own algorithms to deal with double-digit addition and subtraction they develop especially robust understandings of place value. The discussion on pages 473–476 in Bridges Grade 2, Volume 2 describes the instruction in the unit, and shows examples of some of the methods and strategies second graders generate when they are not simply taught to "carry" and "borrow." The samples of students' responses to the Unit 5 Pre- and Post-Assessment on pages 656–658, one of which is shown below, illustrate how students do, in fact, create and revise mathematical models over a period of 6–7 weeks. There are many similar examples throughout the Grade 2 program.

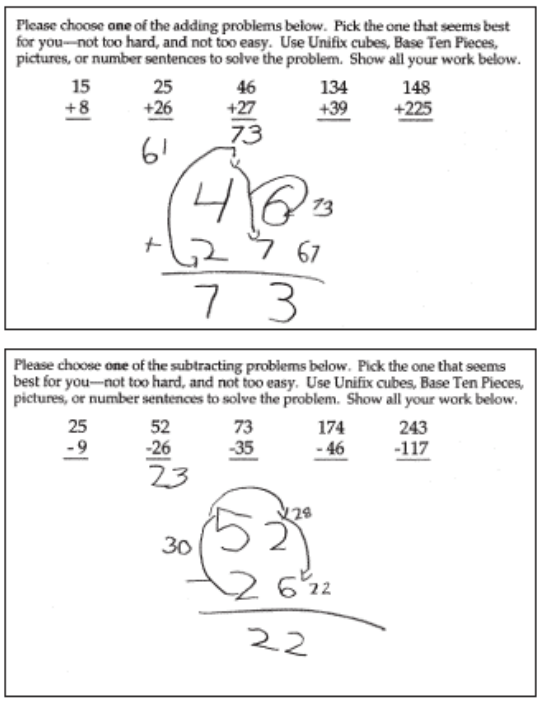


Figure 1, Anna, Session 1

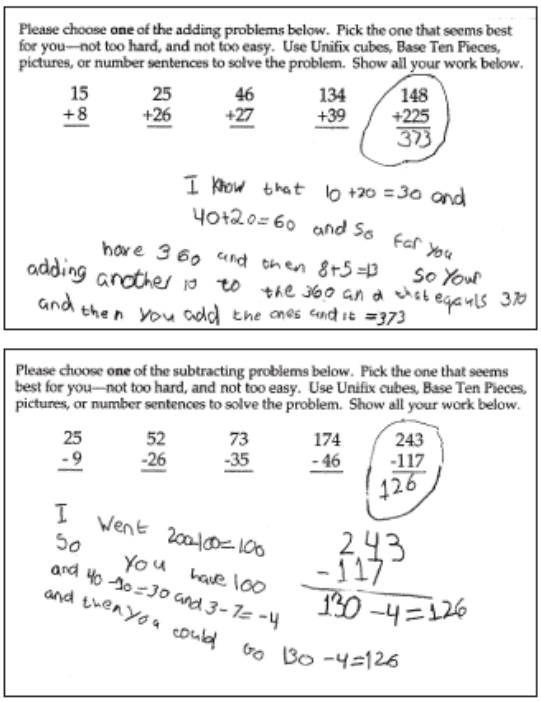


Figure 2, Anna, Session 35

C. Under the heading of Standards for Mathematical Practice 5, reviewers state that, "there is only limited evidence for students choosing tools to problem solve. No evidence was found to support students realizing the strengths and limitations of tools."

If you are willing to widen your view of what constitutes a tool, you'll find that students throughout K-2 are consistently offered the choice of using manipulatives, sketches, numbers, and/or words to both solve problems and explain their solutions and strategies to others. There are variations within each category.

The sketches second graders may use to solve a problem such as 25 – 9 range from individual tally marks to sketches of base ten pieces. The numbers second graders may use to solve the same problem range all the way from using numbers as counters, through a variety of invented algorithms, to the U.S. "standard" algorithm for multi-digit subtraction. The discussion on pages 486–487 in Bridges Grade 2, Volume 2, highlights the fact that students do, in fact, have choices about the tools they use, and points out that the tools our students choose yield information we can use to guide our instruction.

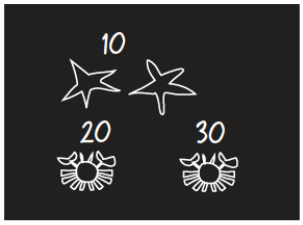
It is the very fact of choice that enables students (with help from a knowledgeable teacher) to move from using less to more efficient and effective tools. If you look at the range of tools and strategies used by first graders to solve the problem

2 sea stars, 2 crabs – how many legs and arms are there in all?

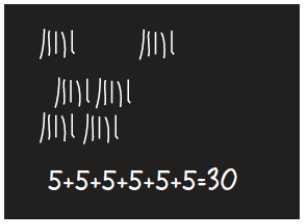
you'll see Unifix cubes counted into stacks of tens and fives, drawings of sea stars and crabs, tally marks, and number sentences, as shown at right in an illustration from Bridges, Grade 1, Volume 1, page 255. We have found over the years that most primary students will move to more efficient strategies/tools as soon as they are able, in effect acknowledging the limitations of such time-intensive and potentially inaccurate strategies as one-by-one counting, tally marks, and so on.

Ollie *I used cubes for the sea stars and crabs. I made stacks of 5 for the sea stars 'cause they have 5 arms, and stacks of 10 for the crabs 'cause they have 10 legs. Then I counted all of the cubes together.*

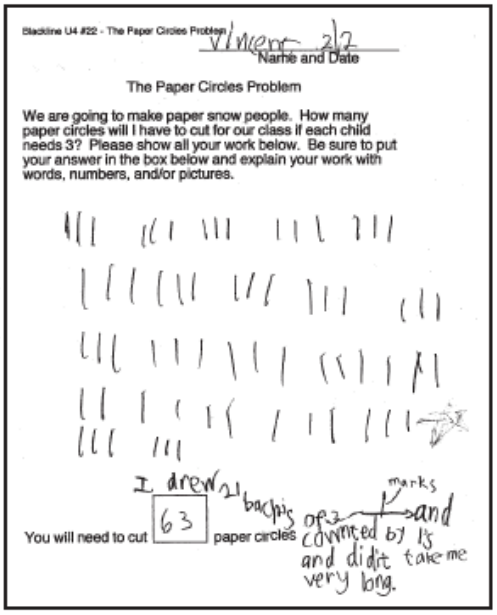
Hillie *I put all their arms and legs on here with pictures. See, here are the sea stars and here are the crabs. I just know that 5 and 5 make 10 and 10 more is 20, and then 30.*



Jessica *I used lines—5 for the sea stars and 10 for the crabs. I put them in 5's and then added them up.*



Here is another example of tool choice and acknowledgement of limitation from Bridges, Grade Two, Unit 4. In Session 22, students are asked to figure out how many paper circles the teacher will need to cut out for everyone in class to make a paper snow person. The materials list specifies the following be provided to students: "Unifix cubes, tile, base ten pieces and/or any other manipulatives children find useful." The illustrations below (real work by real kids) show an enormous range, in terms of the tools selected by students to solve this problem and how those tools were employed. It's subtle, to be sure, but when Vincent writes, "I drew 21 batches of 3 and counted by ones and (it) didn't take me very long" it's a pretty good sign that he's well-aware of the limitations of tally marks as a tool to solve this problem, but doesn't have anything else at his disposal right now.


Blackline U4 #22 - The Paper Circles Problem

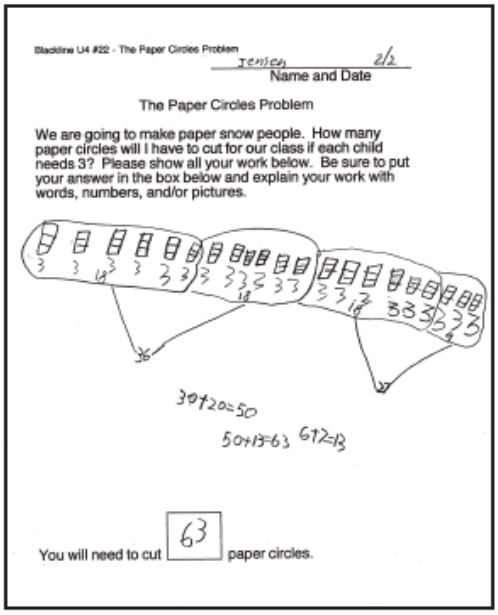
Vincent 2/7
Name and Date

The Paper Circles Problem

We are going to make paper snow people. How many paper circles will I have to cut for our class if each child needs 3? Please show all your work below. Be sure to put your answer in the box below and explain your work with words, numbers, and/or pictures.

I drew 21 batchs of 3 and counted by 1s and didit take me very long.

You will need to cut 63 paper circles


Blackline U4 #22 - The Paper Circles Problem

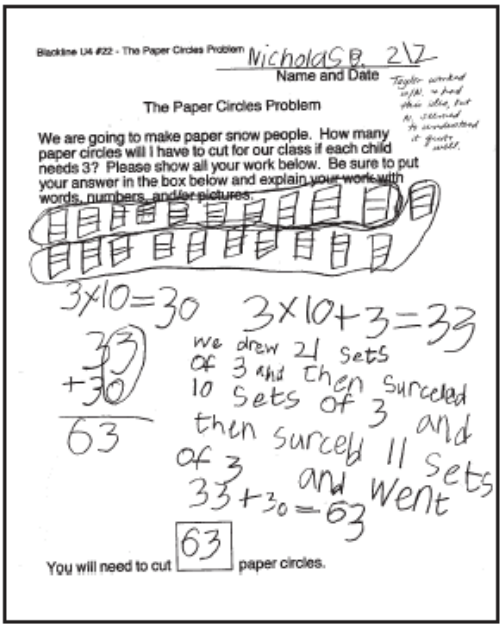
Jensen 2/2
Name and Date

The Paper Circles Problem

We are going to make paper snow people. How many paper circles will I have to cut for our class if each child needs 3? Please show all your work below. Be sure to put your answer in the box below and explain your work with words, numbers, and/or pictures.

30+20=50
50+13=63 63-13

You will need to cut 63 paper circles.


Blackline U4 #22 - The Paper Circles Problem

Nicholas B. 2\7
Name and Date

The Paper Circles Problem

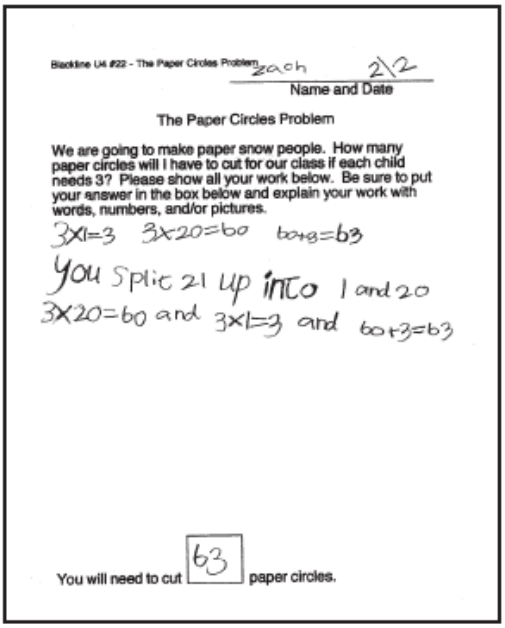
We are going to make paper snow people. How many paper circles will I have to cut for our class if each child needs 3? Please show all your work below. Be sure to put your answer in the box below and explain your work with words, numbers, and/or pictures.

3x10=30 3x10+3=33

33 +30 63

we drew 21 sets of 3 and then surceed 10 sets of 3 then surceed 11 sets of 3 and went 33+30=63

You will need to cut 63 paper circles.


Blackline U4 #22 - The Paper Circles Problem

Zach 2\2
Name and Date

The Paper Circles Problem

We are going to make paper snow people. How many paper circles will I have to cut for our class if each child needs 3? Please show all your work below. Be sure to put your answer in the box below and explain your work with words, numbers, and/or pictures.

3x1=3 3x20=60 60+3=63

You Splic 21 up into 1 and 20 3x20=60 and 3x1=3 and 60+3=63

You will need to cut 63 paper circles.

Although we appreciate the areas identified by the Dana Center evaluation, we believe that Bridges in Mathematics, K-5 exemplifies the mathematical practices advocated by the Common Core State Standards. I would be happy to provide more examples from the program to illustrate the ways in which we meet and exceed standard in the area of mathematical processes: communication, connections to students' everyday lives, problem solving, reasoning, and mathematical representation.

Response to Indiana Teacher Review – Bridges in Mathematics Grades K-2

by Allyn Fisher, Director of Curriculum Development, The Math Learning Center

In looking over the reviews of Bridges in Mathematics K-2, we generally found the notes and ratings from individual reviewers to be fair and thoughtful. However, we are unable to make sense of the summary ratings and related comments.

From a quantitative point of view the individual review sheets for grades K and 1 clearly averaged well above 2 across all categories and yet the final ratings somehow ended up in the 1-2 range. And, across all grades K-2 there seems to be little connection between the individual reviews and summary comments. In the paragraphs below we address these discrepancies and elaborate as to why we strongly disagree with the summary ratings and comments.

> Overall Rating: Weak (1-2)
> Summary/Justification/Evidence: Many portions of this curriculum would be great for supplemental materials and intervention programs. Not good variety of questions and [opportunities] for student inquiry. Don't see spiraling and review.

Bridges in Mathematics was developed with the understanding that students learn at different rates and in different ways. For this reason, there is a great deal of review built in, as well as multiple opportunities, often in different contexts, to learn key skills at each grade level. Number Corner, a key component of the Bridges program, plays an important role in this process. In addition to an hour-long Bridges session each day, teachers conduct a 15–20 minute Number Corner session at another time of the day.

Number Corner is a calendar-based piece that provides a steady stream of basic skills instruction through the year. This enables teachers and students to delve into a particular topic, such as geometry, for a number of weeks, while still keeping an eye on basic facts, place value understandings, time, money, measurement, data, and grade-appropriate computation. Two other features of the Bridges program that provide for consistent practice and review of key skills are the Home Connections homework assignments and the optional Bridges Practice Book for each grade level.

As an example of how the instruction in Bridges flows through an entire year, providing a great deal in the way of spiraling and review, let's look at the approach to teaching basic addition and subtraction facts to 20 in Grade 2. While Unit 1 in the second grade program focuses largely on patterns as central to mathematical thinking, Unit 2 revolves around addition and subtraction story problems. Students solve and then pose a wide variety of problems, ensuring that they understand the operations of addition and subtraction before moving toward fact mastery. During the same time period, students are investigating odd and even numbers during Number Corner, discovering that every even number can be thought of as a double, while every odd number can be thought of as a double plus or minus 1.

With these foundations, as well as what they learned in first grade, second graders move into Unit 3, which is entirely devoted to investigating fact strategies such as doubles,

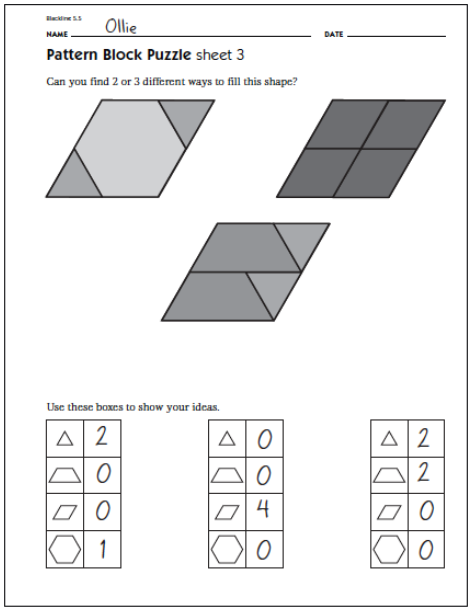
doubles plus or minus 1, adding tens, adding nines, combinations of ten, and so on. Once through this unit, Number Corner and the Bridges Practice Book continue the stream of fact practice through the remainder of the school year, using the Base Ten Bank and the Workout Wheel to revisit addition and subtraction fact strategies, and the Practice Book pages to help develop fluency and speed, as well as provide opportunities to use the facts in the context of many different types of story problems.

If you select any mathematical topic in K-2, whether number to 20 in kindergarten, place value understandings in first grade, or double-digit addition and subtraction in second grade, you will find instruction that flows through the entire year, rather than appearing in a single unit or two, not to be revisited until the following year.

Important Mathematical Ideas: Weak (1-2) Summary/Justification/Evidence: Doesn't give enough opportunity for student inquiry.

We believe that Bridges offers unusually strong opportunities for student inquiry at all three grade levels, and are puzzled by this statement from the reviewers. To begin, Bridges provides 20–30 minutes a day with math centers known as Work Places for first and second graders, as well as kindergartners in full-day programs. Work Places are games and activities designed to provide practice with skills that are currently being taught in the classroom.

In Grade One, Unit 5, for example, there are a total of 12 Work Places offered in conjunction with a month-long geometry unit. These Work Place activities provide students with many different opportunities to identify, sort, build, draw, compose, and decompose 2- and 3-dimensional shapes using a variety of materials. Each Work Place is accompanied by suggestions to teachers for supporting students who may be struggling with a particular concept or challenging students who are ready to go further in their investigations. A simple example is the Pattern Block Puzzles Work Place, in which first graders are asked to use pattern blocks to compose larger 2-dimensional shape "puzzles" in several different ways and record their work, as shown at the right. One of the extensions suggested in Instructional Considerations for this activity is to challenge students to find the fewest number of blocks it takes to build each puzzle. Another extension is to ask students to determine the area of each puzzle as measured in green pattern block triangles.



Ollie I have another idea too. I can use all triangles. I'm going to get another sheet of this same puzzle!

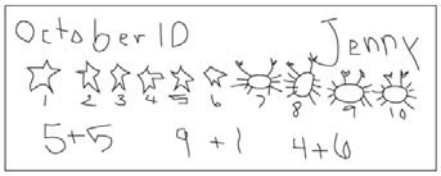
Work Places provide daily intensive, hands-on experiences for students, as well as realistic opportunities for teachers to differentiate, meeting with individuals and small groups to provide support or challenge as needed, as the entire class is productively engaged in mathematically worthwhile tasks. We find that students often develop new variations of games, and with little encouragement from teachers, extend activities into deeper investigations. This is partly facilitated by the fact that the Work Places are a regular feature of the program, and each activity stays in place for several weeks. Thus, if a student is particularly intrigued by a problem or wants to develop a strategy for winning a particular game, he or she can revisit an activity multiple times, deepening his or her engagement with the concepts involved.

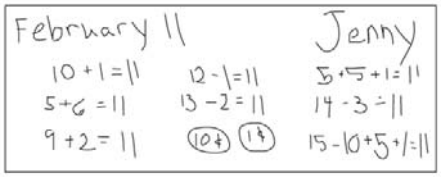
Another feature of Bridges that promotes student engagement and inquiry is a deep commitment to the problem solving process. The lesson design outlined below characterizes many sessions and sets of sessions throughout Bridges and the Number Corner, K-2.

1. A problem is posed.

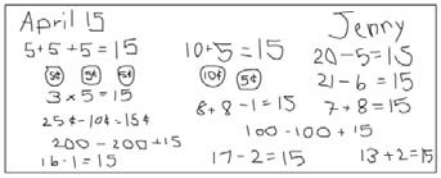
2. The teacher briefly discusses resources and working arrangements. We say little about how the children might go about solving the problem. That's up to them.

3. Children work for an extended period of time. They choose tools—manipulatives, markers, newsprint, and so on—and settle down to work. There are false starts and off-task behavior, to be sure, but soon, most students are constructively engaged.



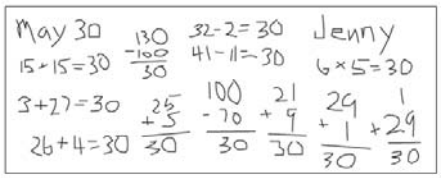


4. The work culminates with a discussion, which has an important role in helping every child learn. Children share their thinking with the class and discuss how they solved the problem.



Many good examples of lessons that follow this design can be found at each grade level. They include:

Bridges, Kindergarten, Volume 2
Sessions 74–83: Posing and Solving Sea Creature Story Problems
Sessions 105–110: Posing and Solving Frog Story Problems

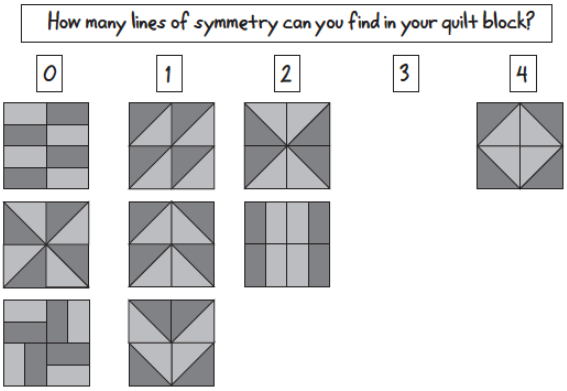


Number Corner, Kindergarten
March Our Month in School: Probability with Frogs & Toads
April Our Month in School: Probability with Cats and Dogs

Bridges, Grade 1, Volume 1, Unit 2
Sessions 4 & 5: Odd & Even Pieces
Sessions 9–14: Crabs and Sea Stars (Investigating Patterns of 10's and 5's)
Sessions 23–25: Posing and Solving Crab & Sea Star Story Problems

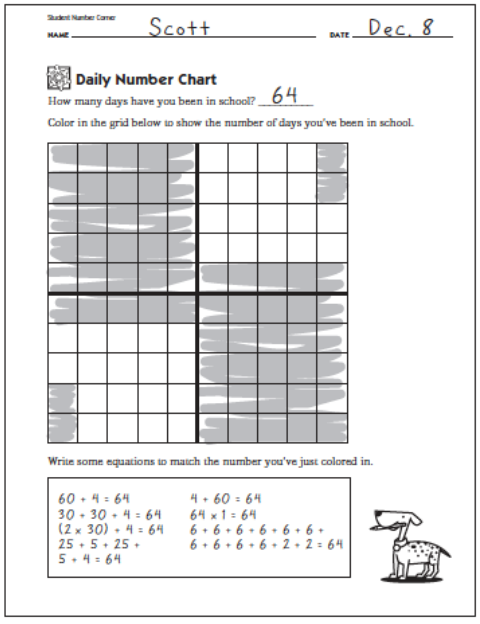
Number Corner, Grade 1, Volumes 1 & 2
Friday's Figuring: Here students are asked to generate observations, representations, expressions, and equations to match the date. This activity is repeated once a week for the entire year. Some weeks the class works together to generate a Friday's Figuring chart. During other weeks, students create their own charts. The display at right shows samples from one student. Notice the changes that appear in the magnitude of the numbers, the operations employed, the notation, and the patterns that appear. The fact that students have an opportunity to revisit this activity throughout the year promotes deep engagement and inquiry, as well as the development of strong number sense.

Bridges, Grade 2, Volume 2, Unit 4
Sessions 2–4: Investigating 3-D Shapes
Sessions 6, 8, 9, 14, 15: Exploring the Concept of Area
Sessions 10–13: Paper Patchwork Quilt Blocks, an Investigation of Tessellation, Symmetry, Shape Relationships, and Fractions
Sessions 22–25: Snow People: Investigating the 3's Number Pattern



Number Corner, Grade 2
November, December, April, May Daily Number Chart: Here, as in Grade 1, students are asked to generate a variety of observations, representations, expressions, and equations to match the number of days they have been in school.

Charts are generated several times a week, sometimes by the class, and sometimes by individual students, promoting deep number sense, and allowing for in-depth student inquiry and investigation.



Skills and Procedures: Weak (1-2) Limited materials for some objectives.

Bridges K-2 was originally written to meet the NCTM Standards. More recently, supplemental activities and grade level practice books were developed to bring Bridges into closer alignment with the NCTM Focal Points and ultimately the Common Core State Standards. Because there is a good degree of overlap between the NCTM Focal Points and the CCSS, especially at grades K–2, Bridges was already well positioned to meet many of the CCSS objectives.

It is also important to note that the supplemental activities for each grade level have been carefully integrated into the existing program rather than simply being appended. An examination of the Common Core Supplement for each grade level will show that activities that don't address Common Core expectations have been stripped out and replaced by activities that do. As we develop new material, it will be added to the grade level supplements, and integrated into the current program by means of revised and updated unit planners, found near the beginning of each CCSS Supplement.

Mathematical Relationships: Weak (1-2) Doesn't require students to make connections between math and real life experiences.

This statement is simply not true, and in fact is directly contradicted by the Dana Center Review conducted for the state of Indiana, where it is noted that, "…this resource is strongly rooted in real-world situations and examples." As classroom teachers, the authors of Bridges were highly aware of the need to, and advantages of, helping primary students make connections between math and their everyday experiences both in and out of the classroom.

The program for each grade level is filled with examples that go far beyond the typical word problems and contrived experiences that are often passed off as real life mathematics. In Kindergarten, this may be best exemplified in the Number Corner, where students count the days of school and the number of children present each day, graph the weather, investigate concepts of time—yesterday, today, tomorrow; morning, noon, and night—in the context of classroom events and daily home and school routines, and eagerly anticipate their birthdays as well as those of their classmates on the calendar grid for each month.

In Grade One, students develop understandings of number to 120 in a variety of ways, including an entire unit devoted to researching penguins. During the course of Unit 4, students investigate six different species of penguins. They use a number line, or number roll similar to the one found in Math Recovery, to measure themselves early in the unit. As children learn about each type of penguin, they measure and cut a length of string to match the height of that penguin, and compare the string to their own height, developing a sense of number as it relates to length. They pile cans of food into a grocery sack to approximate the weight of each penguin, literally developing a sense of how heavy 6 pounds feels, how 8 pounds compares to 15 pounds, and what the numbers on the scale

really mean. The children learn to handle and read thermometers—another form of number lines— as they add ice to water in an attempt to approximate the temperatures of the oceans in which penguins live. They go on to measure, draw, and paint life-sized portraits of their penguins, pose and solve penguin story problems, and in many classrooms, invite their families to view and celebrate their work.

In Grade 2, Unit 6, students develop and apply skills in the areas of measurement, computation, and data by designing and testing marble rolls over a period of several weeks. The unit opens with several days of open exploration, as students are introduced to several variables: ramps of different heights, marbles of different masses, and tubes of different lengths. Engagement and opportunities for student inquiry are strong as youngsters investigate the materials. The first few days are followed by three structured experiments in which students sketch the experimental set-up, write a hypothesis, and conduct and record numerous trials. Later, they average their data, graph the results, draw conclusions, and design new marble rolls on the basis of their findings. This is an opportunity for students to conduct experiments using the same basic processes employed in science labs all over the world.